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Foundations of Query Languages
Summerterm 11
Discussion by 25.05.2011

2. Acyclic Conjunctive Queries and treewidth

Exercise 1 (Acyclic CQ)

a) Prove that a conjunctive query has a join tree iff it is acyclic.

b) Develop a simple polynomial-time algorithm that constructs a join tree from an acyclic conjunctive query.

Exercise 2 (Acyclic CQ)

Consider the following Boolean conjunctive query

$$b \leftarrow a_1(x_1, x_2), a_2(x_2, x_3), \ldots, a_{n-1}(x_{n-1}, x_n), a_n(x_n, x_1).$$

a) Is this query cyclic or acyclic?

b) Let us suppose that each of the attributes has domain $\{1, 2, 3, 4\}$. Furthermore, let us suppose that in our database each of the binary relations contains all of the tuples, such that the attribute values have different parity. E.g. $(1, 2) \in a_1$ (since 1 is odd and 2 even) while $(1, 3) \notin a_1$ (since both 1 and 3 are odd). What is the result of this query if n is odd? (Why? Explain your answer!)

c) Estimate, how much time is required to evaluate this query? (Is it polynomial in n?)

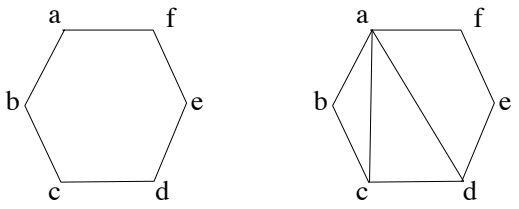
Exercise 3 (Conjunctive query)

The rules of the 4×4 SUDOKU are the following: One has to fill out the empty squares with the numbers $\{1, 2, 3, 4\}$, such that each number occurs only once in each row, in each column and in each 2×2 square surrounded by thick line on the Figure. Formulate the SUDOKU on the Figure as a database problem: Design a relational database schema S, a database D on S, and a query Q, such that $Q(D)$ returns exactly all possible solutions for the SUDOKU game.

	2	3	
1			4
3			2
	1	4	

Exercise 4 (Treewidth)

What is the treewidth of the following graphs? Give a tree decomposition for each graph whose width is the treewidth.



Exercise 5 (Treewidth)
Given a graph $G = (V, E)$, a set of vertices $V' \subseteq V$ is a *vertex cover* iff for every edge $(u, v) \in E$, at least one of u and v belongs to V'. A vertex cover V' is minimal, if there does not exist any vertex cover V'' such that $|V''| < |V'|$. Show that minimal vertex cover of a given graph can be computed in polynomial time if the underlying graph has bounded treewidth. (Hint: design the bottomup processing over the tree nodes on the tree decoposition).